OTHER COLLEGE

FYJC - MATHEMATICS & STATISTICS

HIGHLIGHTS

- ✓ *Solution to all questions*
- ✓ *solutions are put in way the student is expected to reproduce in the exam*
- ✓ *taught in the class room the same way as the solution are put up here . That makes the student to easily go through the solution & prepare him/herself when he/she sits back to revise and recall the topic at any given point of time .*
- ✓ *lastly, if student due to some unavoidable reasons , has missed the lecture , will not have to run here and there to update his/her notes .*
- ✓ *however class room lectures are must for easy passage of understanding & learning the minuest details of the given topic*

PAPER - I

DIFFERENTIATION

Compiled & Conducted @ JKSC

DIFFERENTIATION

DERIVATIVES OF STANDARD FUNCTIONS

$\frac{d}{dx}(k) = 0$

$\frac{d}{dx}(x) = 1$

$\frac{d}{dx}(x^2) = 2x$

$\frac{d}{dx}(x^3) = 3x^2$

$\frac{d}{dx}(x^4) = 4x^3$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

$\frac{d}{dx}\frac{1}{x} = \frac{-1}{x^2}$

$\frac{d}{dx}(\log x) = \frac{1}{x}$

$\frac{d}{dx}(e^x) = e^x$

$\frac{d}{dx}(a^x) = a^x.\log a$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec 2x$

$\frac{d}{dx}(\cot x) = -\text{cosec} 2x$

$\frac{d}{dx}(\sec x) = \sec x \,.\, \tan x$

$\frac{d}{dx}(\text{cosec } x) = -\text{cosec } x \,.\, \cot x$

SUMS ON PRODUCT RULE & QUOTIENT RULE

Q SET -1 : SUMS ON PRODUCT RULE

01. $y = x \,.\, \tan x$

ans : $\frac{dy}{dx} = x \,.\, \sec^2 x + \tan x$

02. $y = x^2 \,.\, \tan x$

ans : $\frac{dy}{dx} = x \,.\, \sec^2 x + 2x.\tan x$

03. $y = e^x \,.\, \tan x$

ans : $\frac{dy}{dx} = e^x \,.\, (\sec^2 x + \tan x)$

04. $y = x^3 \,.\, \cos x$

ans : $\frac{dy}{dx} = 3x^2.\cos x - x^3 \,.\, \sin x$

05. $y = \sin x \,.\, \cos x$

ans : $\frac{dy}{dx} = \cos 2x$

06. $y = x^2 \,.\, 3^x$

ans : $\frac{dy}{dx} = x \,.\, 3^x \,.\, (x.\log 3 + 2)$

07. $y = x^5 \,.\, 5^x$

ans : $\frac{dy}{dx} = x^4 \,.\, 5^x \,.\, (x.\log 5 + 5)$

08. $y = (x^2 + 3x).\log x$

ans : $\frac{dy}{dx} = x + 3 + (2x + 3).\log x$

09. $y = (x^4 + 2x).\sin x$

ans : $\frac{dy}{dx} = (x^4 + 2x).\cos x + (4x^3 + 2).\sin x$

10. $y = (9x^3 - 1).\tan x$

ans : $\frac{dy}{dx} = (9x^3 - 1).\sec^2 x + 27x^2.\tan x$

11. $y = (4x^3 - 7x^2 + 5).\cos x$

ans : $\frac{dy}{dx} = (12x^2 - 14x).\cos x - (4x^3 - 7x^2 + 5).\sin x$

12. $y = x.\sin x + \cos x$

ans : $\frac{dy}{dx} = x.\cos x$

13. $y = 2x.\sin x - x^2.\cos x$

ans : $\frac{dy}{dx} = (x^2 + 2).\sin x$

14. $y = x^4.\log x - e^x.\sin x$

ans : $\frac{dy}{dx} = x^3(1 + 4.\log x) - e^x(\cos x + \sin x)$

Q SET -2 : SUMS ON QUOTIENT RULE

01. $y = \frac{x^3 - 3x + 5}{2x + 1}$

ans : $\frac{dy}{dx} = \frac{4x^3 + 3x^2 - 13}{(2x + 1)^2}$

02. $y = \frac{x^3 + 2x - 1}{x^2 - 2}$

ans : $\frac{dy}{dx} = \frac{2x^4 - 8x^2 + 2x - 4}{(x^2 - 2)^2}$

03. $y = \frac{3x^2 - 4}{x + 5}$

ans : $\frac{dy}{dx} = \frac{3x^2 + 30x + 4}{(x + 5)^2}$

04. $y = \frac{(2x - 1)(3x + 1)}{4x - 1}$

ans : $\frac{dy}{dx} = \frac{24x^2 - 12x + 5}{(4x - 1)^2}$

05. $y = \frac{x + 2}{(x - 3)(x + 4)}$

ans : $\frac{dy}{dx} = \frac{-x^2 - 4x - 14}{(x^2 + x - 12)^2}$

06. $y = \frac{\sqrt{x} + 1}{\sqrt{x} - 1}$

ans : $\frac{dy}{dx} = -\frac{1}{\sqrt{x}(\sqrt{x} - 1)^2}$

07. $y = \frac{3e^x - 2}{x^2 - 4}$

ans : $\frac{dy}{dx} = \frac{e^x(3x^2 - 6x - 12) + 4x}{(x^2 - 4)^2}$

08. $y = \frac{e^x + 2}{(x - 1)(x + 5)}$

ans : $\frac{dy}{dx} = \frac{e^x.(x^2 + 2x - 9) - 4x - 8}{(x^2 + 4x - 5)^2}$

09. $y = \frac{\sin x}{1 + \cos x}$

ans : $\frac{dy}{dx} = \frac{1}{1 + \cos x}$

10. $y = \frac{x + \cos x}{1 + \sin x}$

ans : $\frac{dy}{dx} = \frac{-x.\cos x}{(1 + \sin x)^2}$

11. $y = \frac{1}{\sec x + \tan x}$

ans : $\frac{dy}{dx} = -\frac{1}{1 + \sin x}$

12. $y = \frac{2 + 3.\cos x}{3 + 2.\cos x}$

ans : $\frac{dy}{dx} = \frac{-5.\sin x}{(3 + 2.\cos x)^2}$

13. $y = \dfrac{x^3 - \sin x}{\cos x}$

ans : $\dfrac{dy}{dx} = \dfrac{3x^2.\cos x + x^3.\sin x - 1}{\cos^2 x}$

14. $y = \dfrac{\log x}{x.e^x}$

ans : $\dfrac{dy}{dx} = \dfrac{1 - (x + 1).\log x}{x^2.e^x}$

15. $y = \dfrac{x^2 + 3}{x.\log x}$

ans : $\dfrac{dy}{dx} = \dfrac{(x^2 - 3).\log x - x^2 - 3}{(x.\log x)^2}$

16. $y = \dfrac{x^2 + 2}{x.\log x + 1}$

ans : $\dfrac{dy}{dx} = \dfrac{(x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$

Q SET -3 : SUMS ON COMPOSITE RULE

01. $y = (2x^2 - 5)^4$

ans : $\dfrac{dy}{dx} = 16x(2x^2 - 5)^3$

02. $y = (x^2 - 3)^5$

ans : $\dfrac{dy}{dx} = 10x(x^2 - 3)^4$

03. $y = (5 - x)^3$

ans : $\dfrac{dy}{dx} = -3(5 - x)^2$

04. $y = (3x^2 - 5ax + a^2)^4$

ans : $\dfrac{dy}{dx} = 4(6x - 5a)(3x^2 - 5ax + a^2)^3$

05. $y = (x^2 - 5x + 7)^{3/2}$

ans : $\dfrac{dy}{dx} = \dfrac{3}{2}(x^2 - 5x + 7)^{1/2} . (2x - 5)$

06. $y = (3x^4 - x^3 + 4)^{5/2}$

ans : $\dfrac{dy}{dx} = \dfrac{15x^2}{2}(3x^4 - x^3 + 4)^{3/2}. (4x - 1)$

07. $y = \sqrt{1 + x^2}$

ans : $\dfrac{dy}{dx} = \dfrac{x}{\sqrt{1 + x^2}}$

08. $y = \sqrt{2x^2 + 3x - 4}$

ans : $\dfrac{dy}{dx} = \dfrac{6x^2 + 3}{2\sqrt{2x^2 + 3x - 4}}$

09. $y = \sqrt{5x^2 - 3x + 1}$

ans : $\dfrac{dy}{dx} = \dfrac{10x - 3}{2\sqrt{5x^2 - 3x + 1}}$

10. $y = \sin(3x + 4)$

ans : $\dfrac{dy}{dx} = 3.\cos(3x + 4)$

11. $y = \sec(4x - 3)$

ans : $\dfrac{dy}{dx} = 4.\sec(4x - 3).\tan(4x - 3)$

12. $y = \sec(x^2 + 1)$

ans : $\dfrac{dy}{dx} = 2x . \sec(x^2+1) . \tan(x^2+1)$

13. $y = \sin (2x + 5)^2$

ans : $\frac{dy}{dx} = 4(2x+5) \cdot \cos(2x+5)^2$

14. $y = \tan (3x+2)^2$

ans : $\frac{dy}{dx} = 6(3x+2). \sec^2(3x+2)^2$

15. $y = \tan (5x-3)^2$

ans : $\frac{dy}{dx} = 10.(5x-3). \sec^2(5x-3)^2$

16. $y = \tan (x.e^x)$

ans : $\frac{dy}{dx} = e^x(x + 1) \cdot \sec^2(x.e^x)$

17. $y = \sin^5 x$

ans : $\frac{dy}{dx} = 5. \sin^4 x \cdot \cos x$

18. $y = \sin^2(2x+5)$

ans : $\frac{dy}{dx} = 2 \cdot \sin (4x+10)$

19. $y = \sin^2(x^3)$

ans : $\frac{dy}{dx} = 3x^2 \cdot \sin(2x^3)$

20. $y = e^{2x^2+3}$

ans : $\frac{dy}{dx} = 4x \cdot e^{2x^2+3}$

21. $y = e^{ax^2+bx+c}$

ans : $\frac{dy}{dx} = (2ax + b) \cdot e^{ax^2+bx+c}$

22. $y = e^{(x-1)^3}$

ans : $\frac{dy}{dx} = 3(x-1)^2 \cdot e^{(x-1)^3}$

23. $y = e^{x.\cos x - \sin x}$

ans : $\frac{dy}{dx} = -x \cdot \sin x \cdot e^{x.\cos x - \sin x}$

24 $y = \log (\sin x)$

ans : $\frac{dy}{dx} = \cot x$

25. $y = \log (\sec x + \tan x)$

ans : $\frac{dy}{dx} = \sec x$

26. $y = \log (\operatorname{cosec} x - \cot x)$

ans : $\frac{dy}{dx} = \operatorname{cosec} x$

27. $y = \log (x.\sin x + \cos x)$

ans : $\frac{dy}{dx} = \frac{x.\cos x}{x.\sin x + \cos x}$

28. $y = \log (\sin e^x)$

ans : $\frac{dy}{dx} = e^x \cdot \cot e^x$

29. $y = \log (\tan x)$

ans : $\frac{dy}{dx} = \frac{2}{\sin 2x}$

30. $y = \log (\cos^2 5x)$

ans : $\frac{dy}{dx} = -10 \cdot \cot 5x$

31. $y = \log (\tan 8^x)$

32. $y = 5^{(x^2+1)^3}$

ans : $\frac{dy}{dx} = 6x.(x^2+1)^2 . 5^{(x^2+1)^3} . \log 5$

33. $y = 5^{(x^2-5x+1)^2}$

$\frac{dy}{dx} = 2(x^2-5x+1) .(2x-5). 5^{(x^2-5x+1)^2} . \log 5$

34. $y = 7^{x.\sin x}$

ans : $\frac{dy}{dx} = 7^{x.\sin x} . \log 7 . (x.\cos x + \sin x)$

SOLUTION SET - 1

SUMS ON PRODUCT RULE

01. $y = x \cdot \tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} x$$

$$= x \cdot \sec^2 x + \tan x \cdot (1)$$

$$= x \cdot \sec^2 x + \tan x$$

02. $y = x^2 \cdot \tan x$

$$\frac{dy}{dx} = x^2 \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} x^2$$

$$= x \cdot \sec^2 x + \tan x \cdot (2x)$$

$$= x \cdot \sec^2 x + 2x.\tan x$$

03. $y = e^x \cdot \tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = e^x \frac{d}{dx} \tan x + \tan x \cdot \frac{d}{dx} e^x$$

$$= e^x \cdot \sec^2 x + \tan x \cdot e^x$$

$$= e^x \cdot (\sec^2 x + \tan x)$$

04. $y = x^3 \cdot \cos x$

$$\frac{dy}{dx} = x^3 \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} x^3$$

$$= x^3 \cdot (-\sin x) + \cos x \cdot (3x^2)$$

$$= -x^3 \cdot \sin x + 3x^2 \cdot \cos x$$

$$= 3x^2 \cdot \cos x - x^3 \cdot \sin x$$

05. $y = \sin x \cdot \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = \sin x \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} \sin x$$

$$= \sin x \cdot (-\sin x) + \cos x \cdot \cos x$$

$$= -\sin^2 x + \cos^2 x$$

$$= \cos 2x$$

06. $y = x^2 . 3^x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^2 \frac{d}{dx} 3^x + 3^x . \frac{d}{dx} x^2$$

$$= x^2 . 3^x . \log 3 + 3^x . (2x)$$

$$= 3^x . (x^2 . \log 3 + 2x)$$

$$= x . 3^x . (x.\log 3 + 2)$$

07. $y = x^5 . 5^x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^5 \frac{d}{dx} 5^x + 5^x . \frac{d}{dx} x^5$$

$$= x^5 . 5^x . \log 5 + 5^x . (5x^4)$$

$$= 5^x . (x^5 . \log 5 + 5x^4)$$

$$= x^4 . 5^x . (x.\log 5 + 5)$$

08. $y = (x^2 + 3x) . \log x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (x^2 + 3x) . \frac{d}{dx} \log x + \log x \frac{d}{dx} (x^2 + 3x)$$

$$= (x^2 + 3x) . \frac{1}{x} + \log x . (2x + 3)$$

$$= \frac{x^2 + 3x}{x} + (2x + 3) . \log x$$

$$= x + 3 + (2x + 3) . \log x$$

09. $y = (x^4 + 2x) . \sin x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (x^4 + 2x) . \frac{d}{dx} \sin x + \sin x \frac{d}{dx} (x^4 + 2x)$$

$$= (x^4 + 2x) . \cos x + \sin x . (4x^3 + 2)$$

$$= (x^4 + 2x) . \cos x + (4x^3 + 2) . \sin x$$

10. $y = (9x^3 - 1).\tan x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (9x^3 - 1).\frac{d}{dx}\tan x + \tan x \frac{d}{dx}(9x^3 - 1)$$

$$= (9x^3 - 1).\sec^2 x + \tan x . 27x^2$$

$$= (9x^3 - 1).\sec^2 x + 27x^2 . \tan x$$

11. $y = (4x^3 - 7x^2 + 5).\cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = (4x^3 - 7x^2 + 5).\frac{d}{dx}\cos x + \cos x \frac{d}{dx}(4x^3 - 7x^2 + 5)$$

$$= (4x^3 - 7x^2 + 5) . (-\sin x) + \cos x . (12x^2 - 14x + 0)$$

$$= -(4x^3 - 7x^2 + 5) . \sin x + (12x^2 - 14x) . \cos x$$

$$= (12x^2 - 14x) . \cos x - (4x^3 - 7x^2 + 5) . \sin x$$

12. $y = x . \sin x + \cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x \frac{d}{dx}\sin x + \sin x \frac{d}{dx} x + \frac{d}{dx}\cos x$$

$$= x . \cos x + \sin x .(1) - \sin x$$

$$= x . \cos x + \sin x - \sin x$$

$$= x . \cos x$$

13. $y = 2x.\sin x - x^2.\cos x$

Differentiating wrt x ;

$$\frac{dy}{dx} = 2x \frac{d}{dx}\sin x + \sin x \frac{d}{dx} 2x - \left(x^2 \frac{d}{dx}\cos x + \cos x \frac{d}{dx} x^2 \right)$$

$$= 2x . \cos x + \sin x . 2 - \left(x^2 . (-\sin x) + \cos x . 2x \right)$$

$$= 2x . \cos x + 2. \sin x - \left(- x^2 . \sin x + 2x . \cos x \right)$$

$$= 2x . \cos x + 2. \sin x + x^2 . \sin x - 2x . \cos x$$

$$= 2. \sin x + x^2 . \sin x$$

$$= (x^2 + 2) . \sin x$$

14. $y = x^4 \cdot \log x - e^x \cdot \sin x$

Differentiating wrt x ;

$$\frac{dy}{dx} = x^4 \cdot \frac{d}{dx}\log x + \log x \cdot \frac{d}{dx}x^4 - \left(e^x \cdot \frac{d}{dx}\sin x + \sin x \cdot \frac{d}{dx}e^x\right)$$

$$= x^4 \cdot \frac{1}{x} + \log x \cdot 4x^3 - \left(e^x \cdot \cos x + \sin x \cdot e^x\right)$$

$$= x^3 + 4x^3 \cdot \log x - e^x (\cos x + \sin x)$$

$$= x^3 (1 + 4.\log x) - e^x (\cos x + \sin x)$$

SOLUTION SET - 2

SUMS ON QUOTIENT RULE

01. $y = \dfrac{x^3 - 3x + 5}{2x + 1}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(2x + 1) \cdot \frac{d}{dx}(x^3 - 3x + 5) - (x^3 - 3x + 5)\frac{d}{dx}(2x + 1)}{(2x + 1)^2}$$

$$= \frac{(2x + 1) \cdot (3x^2 - 3) - (x^3 - 3x + 5) \cdot 2}{(2x + 1)^2}$$

$$= \frac{6x^3 - 6x + 3x^2 - 3 - 2.(x^3 - 3x + 5)}{(2x + 1)^2}$$

$$= \frac{6x^3 - 6x + 3x^2 - 3 - 2x^3 + 6x - 10}{(2x + 1)^2}$$

$$= \frac{4x^3 + 3x^2 - 13}{(2x + 1)^2}$$

02. $y = \dfrac{x^3 + 2x - 1}{x^2 - 2}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 - 2) \cdot \frac{d}{dx}(x^3 + 2x - 1) - (x^3 + 2x - 1)\frac{d}{dx}(x^2 - 2)}{(x^2 - 2)^2}$$

$$= \frac{(x^2 - 2).(3x^2 + 2) - (x^3 + 2x - 1) \cdot 2x}{(x^2 - 2)^2}$$

$$= \frac{3x^4 + 2x^2 - 6x^2 - 4 - 2x.(x^3 + 2x - 1)}{(x^2 - 2)^2}$$

$$= \frac{3x^4 + 2x^2 - 6x^2 - 4 - 2x^4 - 4x^2 + 2x}{(x^2 - 2)^2}$$

$$= \frac{2x^4 - 8x^2 + 2x - 4}{(x^2 - 2)^2}$$

03. $y = \dfrac{3x^2 - 4}{x + 5}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x + 5) \cdot \frac{d}{dx}(3x^2 - 4) - (3x^2 - 4)\frac{d}{dx}(x + 5)}{(x + 5)^2}$$

$$= \frac{(x + 5) \cdot 6x - (3x^2 - 4) \cdot 1}{(x + 5)^2}$$

$$= \frac{6x^2 + 30x - 3x^2 + 4}{(x + 5)^2}$$

$$= \frac{3x^2 + 30x + 4}{(x + 5)^2}$$

04. $y = \dfrac{(2x - 1)(3x + 1)}{4x - 1}$

$y = \dfrac{6x^2 - x - 1}{4x - 1}$

Differentiating wrt. x ,

$$\frac{dy}{dx} = \frac{(4x - 1)\frac{d}{dx}(6x^2 - x - 1) - (6x^2 - x - 1)\frac{d}{dx}(4x - 1)}{(4x - 1)^2}$$

$$= \frac{(4x - 1) \cdot (12x - 1) - (6x^2 - x - 1) \cdot 4}{(4x - 1)^2}$$

$$= \frac{48x^2 - 4x - 12x + 1 - 4(6x^2 - x - 1)}{(4x - 1)^2}$$

$$= \frac{48x^2 - 16x + 1 - 24x^2 + 4x + 4}{(4x - 1)^2}$$

$$= \frac{24x^2 - 12x + 5}{(4x - 1)^2}$$

05. $y = \dfrac{x + 2}{(x - 3)(x + 4)}$

$y = \dfrac{x + 2}{x^2 + x - 12}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 + x - 12)\frac{d}{dx}(x + 2) - (x + 2)\frac{d}{dx}(x^2 + x - 12)}{(x^2 + x - 12)^2}$$

$$= \frac{(x^2 + x - 12) \cdot 1 - (x + 2)(2x + 1)}{(x^2 + x - 12)^2}$$

$$= \frac{x^2 + x - 12 - (2x^2 + x + 4x + 2)}{(x^2 + x - 12)^2}$$

$$= \frac{x^2 + x - 12 - (2x^2 + 5x + 2)}{(x^2 + x - 12)^2}$$

$$= \frac{x^2 + x - 12 - 2x^2 - 5x - 2}{(x^2 + x - 12)^2} = \frac{-x^2 - 4x - 14}{(x^2 + x - 12)^2}$$

06. $y = \dfrac{\sqrt{x} + 1}{\sqrt{x} - 1}$

Differentiating wrt x ,

$$\frac{dy}{dx} = \frac{(\sqrt{x} - 1)\frac{d}{dx}(\sqrt{x} + 1) - (\sqrt{x} + 1)\frac{d}{dx}(\sqrt{x} + 1)}{(\sqrt{x} - 1)^2}$$

$$= \frac{(\sqrt{x} - 1)\frac{1}{2\sqrt{x}} - (\sqrt{x} + 1)\frac{1}{2\sqrt{x}}}{(\sqrt{x} - 1)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}\left(\sqrt{x} - 1 - \sqrt{x} - 1\right)}{(\sqrt{x} - 1)^2}$$

$$= \frac{\frac{1}{2\sqrt{x}}(-2)}{(\sqrt{x} - 1)^2}$$

$$= \frac{-1}{\sqrt{x}(\sqrt{x} - 1)^2}$$

07. $y = \dfrac{3e^x - 2}{x^2 - 4}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 - 4)\dfrac{d}{dx}(3e^x - 2) - (3e^x - 2)\dfrac{d}{dx}(x^2 - 4)}{(x^2 - 4)^2}$$

$$= \frac{(x^2 - 4) \cdot 3e^x - (3e^x - 2) \cdot 2x}{(x^2 - 4)^2}$$

$$= \frac{3e^x.(x^2 - 4) - 2x.(3e^x - 2)}{(x^2 - 4)^2}$$

$$= \frac{3e^x.x^2 - 12e^x - 6xe^x + 4x}{(x^2 - 4)^2}$$

$$= \frac{e^x(3x^2 - 6x - 12) + 4x}{(x^2 - 4)^2}$$

08. $y = \dfrac{e^x + 2}{(x - 1)(x + 5)}$

$$y = \frac{e^x + 2}{x^2 + 4x - 5}$$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x^2 + 4x - 5)\dfrac{d}{dx}(e^x + 2) - (e^x + 2)\dfrac{d}{dx}(x^2 + 4x - 5)}{(x^2 + 4x - 5)^2}$$

$$= \frac{(x^2 + 4x - 5) \cdot e^x - (e^x + 2) \cdot (2x + 4)}{(x^2 + 4x - 5)^2}$$

$$= \frac{x^2.e^x + 4x.e^x - 5e^x - (2xe^x + 4e^x + 4x + 8)}{(x^2 + 4x - 5)^2}$$

$$= \frac{x^2.e^x + 4x.e^x - 5e^x - 2xe^x - 4e^x - 4x - 8}{(x^2 + 4x - 5)^2}$$

$$= \frac{e^x \cdot (x^2 + 4x - 5 - 2x - 4) - 4x - 8}{(x^2 + 4x - 5)^2}$$

$$= \frac{e^x \cdot (x^2 + 2x - 9) - 4x - 8}{(x^2 + 4x - 5)^2}$$

09. $y = \dfrac{\sin x}{1 + \cos x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \cos x)\frac{d}{dx}\sin x - \sin x \frac{d}{dx}(1 + \cos x)}{(1 + \cos x)^2}$$

$$= \frac{(1 + \cos x) \cdot \cos x - \sin x (0 - \sin x)}{(1 + \cos x)^2}$$

$$= \frac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$$

$$= \frac{\cos x + 1}{(1 + \cos x)^2}$$

$$= \frac{1}{1 + \cos x}$$

10. $y = \dfrac{x + \cos x}{1 + \sin x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \sin x)\frac{d}{dx}(x + \cos x) - (x + \cos x)\frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x) \cdot (1 - \sin x) - (x + \cos x)(0 + \cos x)}{(1 + \sin x)^2}$$

$$= \frac{1 - \sin^2 x - (x + \cos x) \cdot \cos x}{(1 + \sin x)^2}$$

$$= \frac{\cos^2 x - x.\cos x - \cos^2 x}{(1 + \sin x)^2}$$

$$= \frac{-x \cdot \cos x}{(1 + \sin x)^2}$$

11. $y = \dfrac{1}{\sec x + \tan x}$

$y = \dfrac{\cos x}{1 + \sin x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(1 + \sin x)\frac{d}{dx}\cos x - \cos x \frac{d}{dx}(1 + \sin x)}{(1 + \sin x)^2}$$

$$= \frac{(1 + \sin x) . (-\sin x) - \cos x (0 + \cos x)}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - \sin^2 x - \cos^2 x}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - (\sin^2 x + \cos^2 x)}{(1 + \sin x)^2}$$

$$= \frac{-\sin x - 1}{(1 + \sin x)^2}$$

$$= \frac{-(1 + \sin x)}{(1 + \sin x)^2} = \frac{-1}{1 + \sin x}$$

12. $y = \dfrac{2 + 3.\cos x}{3 + 2.\cos x}$

$$\frac{dy}{dx} = \frac{(3 + 2.\cos x)\frac{d}{dx}(2 + 3.\cos x) - (2 + 3.\cos x)\frac{d}{dx}(3 + 2.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x) . \left(0 + 3(-\sin x)\right) - (2 + 3.\cos x)\left(0 + 2(-\sin x)\right)}{(3 + 2.\cos x)^2}$$

$$= \frac{(3 + 2.\cos x)(-3.\sin x) - (2 + 3.\cos x).(-2.\sin x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-3.\sin x (3 + 2.\cos x) + 2.\sin x (2 + 3.\cos x)}{(3 + 2.\cos x)^2}$$

$$= \frac{-9.\sin x - 6.\sin x . \cos x + 4.\sin x + 6.\sin x .\cos x}{(3 + 2.\cos x)^2}$$

$$= \frac{-5.\sin x}{(3 + 2.\cos x)^2}$$

13. $y = \dfrac{x^3 - \sin x}{\cos x}$

Differentiating wrt x

$$\frac{dy}{dx} = \frac{\cos x \frac{d}{dx}(x^3 - \sin x) - (x^3 - \sin x)\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$= \frac{\cos x \,.\, (3x^2 - \cos x) - (x^3 - \sin x)\,.\,(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos x \,.\, (3x^2 - \cos x) + \sin x \,.\, (x^3 - \sin x)}{\cos^2 x}$$

$$= \frac{3x^2.\cos x - \cos^2 x + x^3 \,.\, \sin x - \sin^2 x}{\cos^2 x}$$

$$= \frac{3x^2.\cos x + x^3.\sin x - (\sin^2 x + \cos^2 x)}{\cos^2 x}$$

$$= \frac{3x^2.\cos x + x^3.\sin x - 1}{\cos^2 x}$$

14. $y = \dfrac{\log x}{x.e^x}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{x.e^x \frac{d}{dx}\log x - \log x \frac{d}{dx}(x.e^x)}{(x.e^x)^2}$$

$$= \frac{x.e^x \,.\, \frac{1}{x} - \log x \left(x \,.\, \frac{d}{dx} e^x + e^x \frac{d}{dx} x\right)}{(x.e^x)^2}$$

$$= \frac{e^x - \log x \, x.e^x + e^x \,.1}{(x.e^x)^2}$$

$$= \frac{e^x - \log x \,.\, e^x (x + 1)}{(x.e^x)^2}$$

$$= \frac{e^x \left(1 - (x + 1).\log x\right)}{x^2.(e^x)^2}$$

$$= \frac{1 - (x + 1).\log x}{x^2.e^x}$$

15. $y = \dfrac{x^2 + 3}{x.\log x}$, Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{x.\log x \frac{d}{dx}(x^2 + 3) - (x^2 + 3)\frac{d}{dx}(x.\log x)}{(x.\log x)^2}$$

$$= \frac{x.\log x \,.\, 2x - (x^2 + 3).\left(x\frac{d}{dx}\log x + \log x \frac{d}{dx}x\right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + 3)\left(x\frac{1}{x} + \log x \,.\, 1\right)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + 3).(1 + \log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - (x^2 + x^2.\log x + 3 + 3\log x)}{(x.\log x)^2}$$

$$= \frac{2x^2.\log x - x^2 - x^2.\log x - 3 - 3\log x}{(x.\log x)^2}$$

$$= \frac{(2x^2 - x^2 - 3)\log x - x^2 - 3}{(x.\log x)^2}$$

$$= \frac{(x^2 - 3).\log x - x^2 - 3}{(x.\log x)^2}$$

16. $y = \dfrac{x^2 + 2}{x.\log x + 1}$

Differentiating wrt x ;

$$\frac{dy}{dx} = \frac{(x.\log x + 1)\frac{d}{dx}(x^2 + 2) - (x^2 + 2)\frac{d}{dx}(x.\log x + 1)}{(x.\log x + 1)^2}$$

$$= \frac{(x.\log x + 1) \,.\, 2x - (x^2 + 2).\left(x\frac{d}{dx}\log x + \log x \frac{d}{dx}x + 0\right)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + 2)\left(x\frac{1}{x} + \log x \,.\, 1\right)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + 2).(1 + \log x)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - (x^2 + x^2.\log x + 2 + 2\log x)}{(x.\log x + 1)^2}$$

$$= \frac{2x^2.\log x + 2x - x^2 - x^2.\log x - 2 - 2\log x}{(x.\log x + 1)^2}$$

$$= \frac{(2x^2 - x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$$

$$= \frac{(x^2 - 2).\log x + 2x - x^2 - 2}{(x.\log x + 1)^2}$$

SOLUTION SET - 3

QSET 3 : SUMS ON COMPOSITE RULE

01. $y = (2x^2 - 5)^4$

Differentiate wrt x ;

$$\frac{dy}{dx} = 4(2x^2 - 5)3 \cdot \frac{d}{dx}(2x^2 - 5)$$

$$= 4(2x^2 - 5)^3 \cdot (4x - 0)$$

$$= 16x(2x^2 - 5)^3$$

02. $y = (x^2 - 3)^5$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5(x^2 - 3)^4 \cdot \frac{d}{dx}(x^2 - 3)$$

$$= 5(x^2 - 3)^4 \cdot (2x - 0)$$

$$= 10x(x^2 - 3)^4$$

03. $y = (5 - x)^3$

Differentiate wrt x ;

$$\frac{dy}{dx} = 3(5 - x)^2 \frac{d}{dx}(5 - x)$$

$$= 3(5 - x)^2 .(0 - 1)$$

$$= -3(5 - x)^2$$

04. $y = (3x^2 - 5ax + a^2)^4$

Differentiate wrt x ;

$$\frac{dy}{dx} = 4(3x^2 - 5ax + a^2)^3 \frac{d}{dx}(3x^2 - 5ax + a^2)$$

$$= 4(3x^2 - 5ax + a^2)^3 . (6x - 5a + 0)$$

$$= 4(6x - 5a)(3x^2 - 5ax + a^2)^3$$

05. $y = (x^2 - 5x + 7)^{3/2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{3}{2}(x^2 - 5x + 7)^{3/2-1} \frac{d}{dx}(x^2 - 5x + 7)$$

$$= \frac{3}{2}(x^2 - 5x + 7)^{1/2} \cdot (2x - 5 + 0)$$

$$= \frac{3}{2}(x^2 - 5x + 7)^{1/2} \cdot (2x - 5)$$

06. $y = (3x^4 - x^3 + 4)^{5/2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{5}{2}(3x^4 - x^3 + 4)^{5/2-1} \frac{d}{dx}((3x^4 - x^3 + 4)$$

$$= \frac{5}{2}(3x^4 - x^3 + 4)^{3/2} \cdot (12x^3 - 3x^2)$$

$$= \frac{5}{2}(3x^4 - x^3 + 4)^{3/2} \cdot 3x^2(4x - 1)$$

$$= \frac{15x^2}{2}(3x^4 - x^3 + 4)^{3/2} \cdot (4x - 1)$$

07. $y = \sqrt{1 + x^2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{2\sqrt{1 + x^2}} \cdot \frac{d}{dx}(1 + x^2)$$

$$= \frac{1}{2\sqrt{1 + x^2}} \cdot 2x$$

$$= \frac{x}{\sqrt{1 + x^2}}$$

08. $y = \sqrt{2x^2 + 3x - 4}$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{2\sqrt{2x^2 + 3x - 4}} \frac{d}{dx}(2x^3 + 3x - 4)$$

$$= \frac{1}{2\sqrt{2x^2 + 3x - 4}} \cdot (6x^2 + 3)$$

$$= \frac{6x^2 + 3}{2\sqrt{2x^2 + 3x - 4}}$$

09. $y = \sqrt{5x^2 - 3x + 1}$

$$\frac{dy}{dx} = \frac{1}{2\sqrt{5x^2 - 3x + 1}} \frac{d}{dx}(5x^2 - 3x + 1)$$

$$= \frac{1}{2\sqrt{5x^2 - 3x + 1}} \cdot (10x - 3)$$

$$= \frac{10x - 3}{2\sqrt{5x^2 - 3x + 1}}$$

10. $y = \sin(3x + 4)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \cos(3x + 4) \cdot \frac{d}{dx}(3x + 4)$$

$$= \cos(3x + 4) \cdot 3$$

$$= 3\cos(3x + 4)$$

11. $y = \sec(4x - 3)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \sec(4x - 3) \cdot \tan(4x - 3) \cdot \frac{d}{dx}(4x - 3)$$

$$= \sec(4x - 3) \cdot \tan(4x - 3) \cdot 4$$

$$= 4 \cdot \sec(4x - 3) \cdot \tan(4x - 3)$$

12. $y = \sec(x^2 + 1)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \sec(x^2+1) \cdot \tan(x^2+1) \cdot \frac{d}{dx}(x^2+1)$$

$$= \sec(x^2+1) \cdot \tan(x^2+1) \cdot 2x$$

$$= 2x \cdot \sec(x^2+1) \cdot \tan(x^2+1)$$

13. $y = \sin(2x + 5)^2$

Differentiate wrt x ;

$$\frac{dy}{dx} = \cos(2x+5)^2 \frac{d}{dx}(2x+5)^2$$

$$= \cos(2x+5)^2 \cdot 2(2x+5) \frac{d}{dx}(2x+5)$$

$$= \cos(2x+5)^2 \cdot 2(2x+5) \cdot 2$$

$$= 4(2x+5) \cdot \cos(2x+5)^2$$

14. $y = \tan (3x+2)^2$

$\frac{dy}{dx} = \sec^2(3x+2)^2 \frac{d}{dx}(3x+2)^2$

$= \sec^2(3x+2)^2 \cdot 2(3x+2) \frac{d}{dx}(3x+2)$

$= \sec^2(3x+2)^2 \cdot 2(3x+2) \cdot 3$

$= 6(3x+2). \sec^2(3x+2)^2 .$

15. $y = \tan (5x-3)^2$

$\frac{dy}{dx} = \sec^2(5x-3)^2 \frac{d}{dx}(5x-3)^2$

$= \sec^2(5x-3)^2 \cdot 2(5x-3) \frac{d}{dx}(5x-3)$

$= \sec^2(5x-3)^2 \cdot 2(5x-3) \cdot 5$

$= 10.(5x-3). \sec^2(5x-3)^2 .$

16. $y = \tan (x.e^x)$

$\frac{dy}{dx} = \sec^2(x.e^x) \cdot \frac{d}{dx}(x.e^x)$

$= \sec^2(x.e^x) \left(x \frac{d}{dx} e^x + e^x \frac{d}{dx} x \right)$

$= \sec^2(x.e^x) .(x e^x + e^x .1)$

$= \sec^2(x.e^x) . e^x(x + 1)$

$= e^x(x + 1) . \sec^2(x.e^x)$

17. $y = \sin^5 x$

$\frac{dy}{dx} = 5.\sin^4 x . \frac{d}{dx} \sin x$

$= 5. \sin^4 x . \cos x$

18. $y = \sin^2(2x+5)$

Differentiate wrt x ;

$\frac{dy}{dx} = 2. \sin(2x+5) . \frac{d}{dx} \sin (2x+5)$

$= 2. \sin(2x+5) .\cos (2x+5) \frac{d}{dx}(2x+5)$

$= 2. \sin(2x+5) . \cos (2x+5) . 2$

$= \sin (4x+10) . 2$

$= 2 . \sin (4x+10)$

19. $y = \sin^2(x^3)$

Differentiate wrt x ;

$\frac{dy}{dx} = 2.\sin(x^3) . \frac{d}{dx} \sin(x^3)$

$= 2.\sin(x^3) . \cos(x^3) \frac{d}{dx}(x^3)$

$= 2.\sin(x^3) . \cos(x^3) . 3x^2$

$= \sin (2x^3) . 3x^2$

$= 3x^2 . \sin(2x^3)$

20. $y = e^{2x^2+3}$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{2x^2+3} \cdot \frac{d}{dx}(2x^2 + 3)$$

$$= e^{2x^2+3} \cdot 4x$$

$$= 4x \cdot e^{2x^2+3}$$

21. $y = e^{ax^2+bx+c}$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{ax^2+bx+c} \cdot \frac{d}{dx}(ax^2+bx+c)$$

$$= e^{ax^2+bx+c} \cdot (2ax + b)$$

$$= (2ax + b) \cdot e^{ax^2+bx+c}$$

22. $y = e^{(x-1)^3}$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{(x-1)^3} \cdot \frac{d}{dx}(x-1)^3$$

$$= e^{(x-1)^3} \cdot 3(x-1)^2 \cdot \frac{d}{dx}(x - 1)$$

$$= 3(x-1)^2 \cdot e^{(x-1)^3}$$

23. $y = e^{x.\cos x - \sin x}$

Differentiate wrt x ;

$$\frac{dy}{dx} = e^{x.\cos x - \sin x} \cdot \frac{d}{dx}(x.\cos x - \sin x)$$

$$= e^{x.\cos x - \sin x} \cdot \left(x \frac{d}{dx}\cos x + \cos x \frac{d}{dx} x - \frac{d}{dx}\sin x \right)$$

$$= e^{x.\cos x - \sin x} \cdot \left(x.(-\sin x) + \cos x .1 - \cos x \right)$$

$$= e^{x.\cos x - \sin x} \cdot (-x . \sin x + \cos x - \cos x)$$

$$= -x . \sin x \cdot e^{x.\cos x - \sin x}$$

24. $y = \log(\sin x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\sin x}\frac{d}{dx}(\sin x)$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

25. $y = \log(\sec x + \tan x)$

Differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{\sec x + \tan x}\frac{d}{dx}(\sec x + \tan x)$$

$$= \frac{1}{\sec x + \tan x}.(\sec x.\tan x + \sec^2 x)$$

$$= \frac{\sec x(\sec x + \tan x)}{\sec x + \tan x}$$

$$= \sec x$$

26. $y = \log(\operatorname{cosec} x - \cot x)$

differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\operatorname{cosec} x - \cot x}\frac{d}{dx}(\operatorname{cosec} x - \cot x)$$

$$= \frac{(-\operatorname{cosec} x.\cot x + \operatorname{cosec}^2 x)}{\operatorname{cosec} x - \cot x}$$

$$= \frac{\operatorname{cosec} x(\operatorname{cosec} x - \cot x)}{\operatorname{cosec} x - \cot x}$$

$$= \operatorname{cosec} x$$

27. $y = \log(x.\sin x + \cos x)$

differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{x.\sin x + \cos x}\frac{d}{dx}(x.\sin x + \cos x)$$

$$= \frac{1}{x.\sin x + \cos x}\left(x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}x - \sin x\right)$$

$$= \frac{1}{x.\sin x + \cos x}(x.\cos x + \sin x - \sin x)$$

$$= \frac{x.\cos x}{x.\sin x + \cos x}$$

28. $y = \log(\sin e^x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\sin e^x}\frac{d}{dx}(\sin e^x)$$

$$= \frac{1}{\sin e^x}.\cos e^x.\frac{d}{dx}e^x$$

$$= \frac{\cos e^x}{\sin e^x}.e^x$$

$$= e^x.\cot e^x$$

29. $y = \log(\tan x)$

Differentiate wrt x

$$\frac{dy}{dx} = \frac{1}{\tan x}\frac{d}{dx}\tan x$$

$$= \frac{1}{\tan x}.\sec^2 x$$

$$= \frac{1}{\frac{\sin x}{\cos x}}.\frac{1}{\cos^2 x}$$

$$= \frac{2}{2\sin x.\cos x} = \frac{2}{\sin 2x}$$

30. $y = \log(\cos^2 5x)$

$y = 2.\log(\cos 5x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = 2\,\frac{1}{\cos 5x}\,\frac{d}{dx}\cos 5x$$

$$= 2\,\frac{1}{\cos 5x}(-\sin 5x)\,\frac{d}{dx}5x$$

$$= 2\,\frac{1}{\cos 5x}(-\sin 5x)\,.\,5$$

$$= -10\,.\cot 5x$$

31. $y = \log(\tan 8^x)$

Differentiate wrt x ;

$$\frac{dy}{dx} = \frac{1}{\tan 8^x}\,\frac{d}{dx}(\tan 8^x)$$

$$= \frac{1}{\tan 8^x}\,.\,\sec^2 8^x\,.\,\frac{d}{dx}8^x$$

$$= \frac{1}{\tan 8^x}\,.\sec^2 8^x\,.\,8^x\,.\log 8$$

$$= \frac{1}{\frac{\sin 8^x}{\cos 8^x}}\,.\,\frac{1}{\cos^2 8^x}\,.\,8^x.\log 8$$

$$= \frac{2}{2.\sin 8^x.\cos 8^x}\,.\,8^x.\log 8$$

$$= \frac{2.8^x.\log 8}{\sin(2.8^x)}$$

32. $y = 5^{(x^2+1)^3}$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5^{(x^2+1)^3}\,.\log 5\,.\,\frac{d}{dx}(x^2+1)^3$$

$$= 5^{(x^2+1)^3}\,.\log 5\,.\,3(x^2+1)^2.\,\frac{d}{dx}(x^2+1)$$

$$= 5^{(x^2+1)^3}\,.\log 5\,.\,3(x^2+1)^2.\,2x$$

$$= 6x.(x^2+1)^2\,.\,5^{(x^2+1)^3}\,.\log 5$$

33. $y = 5^{(x^2-5x+1)^2}$

Differentiate wrt x ;

$$\frac{dy}{dx} = 5^{(x^2-5x+1)^2}\,.\log 5\,.\,\frac{d}{dx}(x^2-5x+1)^2$$

$$= 5^{(x^2-5x+1)^2}\,.\log 5.\,2(x^2-5x+1)\,.\frac{d}{dx}(x^2-5x+1)$$

$$= 5^{(x^2-5x+1)^2}\,.\log 5\,.\,2(x^2-5x+1)\,.(2x-5)$$

$$= 2(x^2-5x+1)\,.(2x-5)\,.\,5^{(x^2-5x+1)^2}\,.\log 5$$

34. $y = 7^{x.\sin x}$

$$\frac{dy}{dx} = 7^{x.\sin x} . \log 7 . \frac{d}{dx} x.\sin x$$

$$= 7^{x.\sin x} . \log 7 . \left(x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x \right)$$

$$= 7^{x.\sin x} . \log 7 . (x.\cos x + \sin x)$$